The Radiance Equation

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Introduction

- **Lighting** is the central problem of real-time graphics rendering
  - Arbitrary shaped lights
  - Changes in lighting conditions
  - Real-time shadows
  - Real-time reflections
  - Mixtures of many different types of surface
Introduction

- **Real-time walkthrough with global illumination**
  - Possible under limited conditions
    - Radiosity (diffuse surfaces only)

- **Real-time interaction**
  - Not possible except for special case local illumination

- **Why is the problem so hard?**
Visible light is electromagnetic radiation with wavelengths approximately in the range from 400nm to 700nm.
Light: Photons

- Light can be viewed as wave or particle phenomenon
- Particles are photons
  - packets of energy which travel in a straight line in vacuum with velocity $c$ (300,000 km.p.s.)
- The problem of how light interacts with surfaces in a volume of space is an example of a transport problem.
Light: Radiant Power

- $\Phi$ denotes the *radian energy* or *flux* in a volume $V$.

- The flux is the *rate of energy flowing* through a surface per unit time (watts).

- The energy is proportional to the particle flow, since each photon carries energy.

- The flux may be thought of as the *flow of photons* per unit time.
Light: Flux Equilibrium

- **Total flux in a volume in dynamic equilibrium**
  - Particles are flowing
  - Distribution is constant
- **Conservation of energy**
  - Total *energy input* into the volume = total energy that is *output* by or absorbed by matter within the volume.
Light: Fundamental Equation

- **Input**
  - Emission – emitted from within volume
  - Inscattering – flows from outside

- **Output**
  - Streaming – without interaction with matter in the volume
  - Outscattering – reflected out from matter
  - Absorption – by matter within the volume

- **Input = Output**
Light: Equation

- $\Phi(p, \omega)$ denotes flux at $p \in V$, in direction $\omega$
- It is possible to write down an integral equation for $\Phi(p, \omega)$ based on:
  - Emission+Inscattering = Streaming+Outscattering + Absorption
- Complete knowledge of $\Phi(p, \omega)$ provides a complete solution to the graphics rendering problem.
- Rendering is about solving for $\Phi(p, \omega)$. 
Simplifying Assumptions

- **Wavelength independence**
  - No interaction between wavelengths (no *fluorescence*)

- **Time invariance**
  - Solution remains valid over time unless scene changes (no *phosphorescence*)

- **Light transports in a vacuum** (non-participating medium) –
  - ‘free space’ – interaction only occurs at the surfaces of objects
Radiance

Radiance \((L)\) is the flux that leaves a surface, per unit projected area of the surface, per unit solid angle of direction.

\[
d\Phi = L \, dA \cos\theta \, d\omega
\]
Radiance

- For computer graphics the basic particle is not the photon and the energy it carries but the ray and its associated radiance.

Radiance is constant along a ray.
Radiance: Radiosity, Irradiance

- **Radiosity** - is the flux per unit area that radiates from a surface, denoted by $B$.
  - $\text{d}\Phi = B \text{ d}A$

- **Irradiance** is the flux per unit area that arrives at a surface, denoted by $E$.
  - $\text{d}\Phi = E \text{ d}A$
Reflectance

- BRDF
  - Bi-directional
  - Reflectance
  - Distribution
  - Function

- Relates
  - Reflected radiance to incoming irradiance

\[ f(p, \omega_i, \omega_r) \]

Illustration:
- Incident ray \( \omega_i \)
- Reflected ray \( \omega_r \)
- Illumination hemisphere
Reflectance: BRDF

- Reflected Radiance = BRDF × Irradiance
  \[ L(p, \omega_r) = f(p, \omega_i, \omega_r) E(p, \omega_i) \]
  \[ = f(p, \omega_i, \omega_r) L(p, \omega_i) \cos \theta_i \, d\omega_i \]

- In practice BRDF’s hard to specify
- Rely on ideal types
  - Perfectly **diffuse** reflection
  - Perfectly **specular** reflection
  - Glossy reflection
- BRDFs taken as **additive mixture** of these
The Radiance Equation

Radiance $L(p, \omega)$ at a point $p$ in direction $\omega$ is the sum of
- Emitted radiance $L_e(p, \omega)$
- Total reflected radiance

Radiance = Emitted Radiance + Total Reflected Radiance
The Radiance Equation: Reflection

- Total reflected radiance in direction $\omega$:
  \[ \int f(p, \omega_i, \omega) \, L(p, \omega_i) \cos \theta_i \, d\omega_i \]

- Radiance Equation:
  \[ L(p, \omega) = L_e(p, \omega) + \int f(p, \omega_i, \omega) \, L(p, \omega_i) \cos \theta_i \, d\omega_i \]
  
  (Integration over the illumination hemisphere)
The Radiance Equation

- $p$ is considered to be on a surface, but can be anywhere, since radiance is constant along a ray, trace back until surface is reached at $p^*$, then
  \[ L(p, \omega_i) = L(p^*, \omega_i) \]

$L(p, \omega)$ depends on all $L(p^*, \omega_i)$ which in turn are recursively defined.

The radiance equation models global illumination.
Traditional Solutions to the Radiance Equation

- The radiance equation embodies totality of all 2D projections (view).
- Extraction of a 2D projection to form an image is called rendering.
Traditional Solutions

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<td>View Independent</td>
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(Image Based Rendering)

- IBR not a ‘traditional’ solution
- Images for a new view constructed from a large collection of existing images
- No lighting computations at all.
- Light Field Rendering specific instance to be discussed later.
Visibility

- Where does an incident ray through the image plane come from?
  - Which surface?
- Ray tracing in principle has to search all surfaces for possible intersections
- Radiosity has to include visibility in form-factor calculations between surfaces
- Real-time rendering solves visibility problem on a pixel by pixel basis (z-buffer).
  - Major complication for large scenes
- We will see later that LFR does not have this visibility problem.
Conclusions

- Graphics rendering is concerned with solution of integral radiance equation.
- Traditional solutions are various kinds of approximations to this equation.
- Rendering is the process of extracting images from the equation.
- Rendering may be view dependent or independent, together with a global or local illumination solution.